ANALYTIC GEOMETRY

EQUATION OF A LINE

$$y = a \cdot x + b$$

SLOPE
(RATE OF CHANGE)

Y-INT
(INITIAL VALUE)

SLOPE
(RATE OF CHANGE)

SLOPE (a) $= \dfrac{\text{RISE}}{\text{RUN}} = \dfrac{Y_2 - Y_1}{X_2 - X_1}$

HOW TO BUILD A RULE...

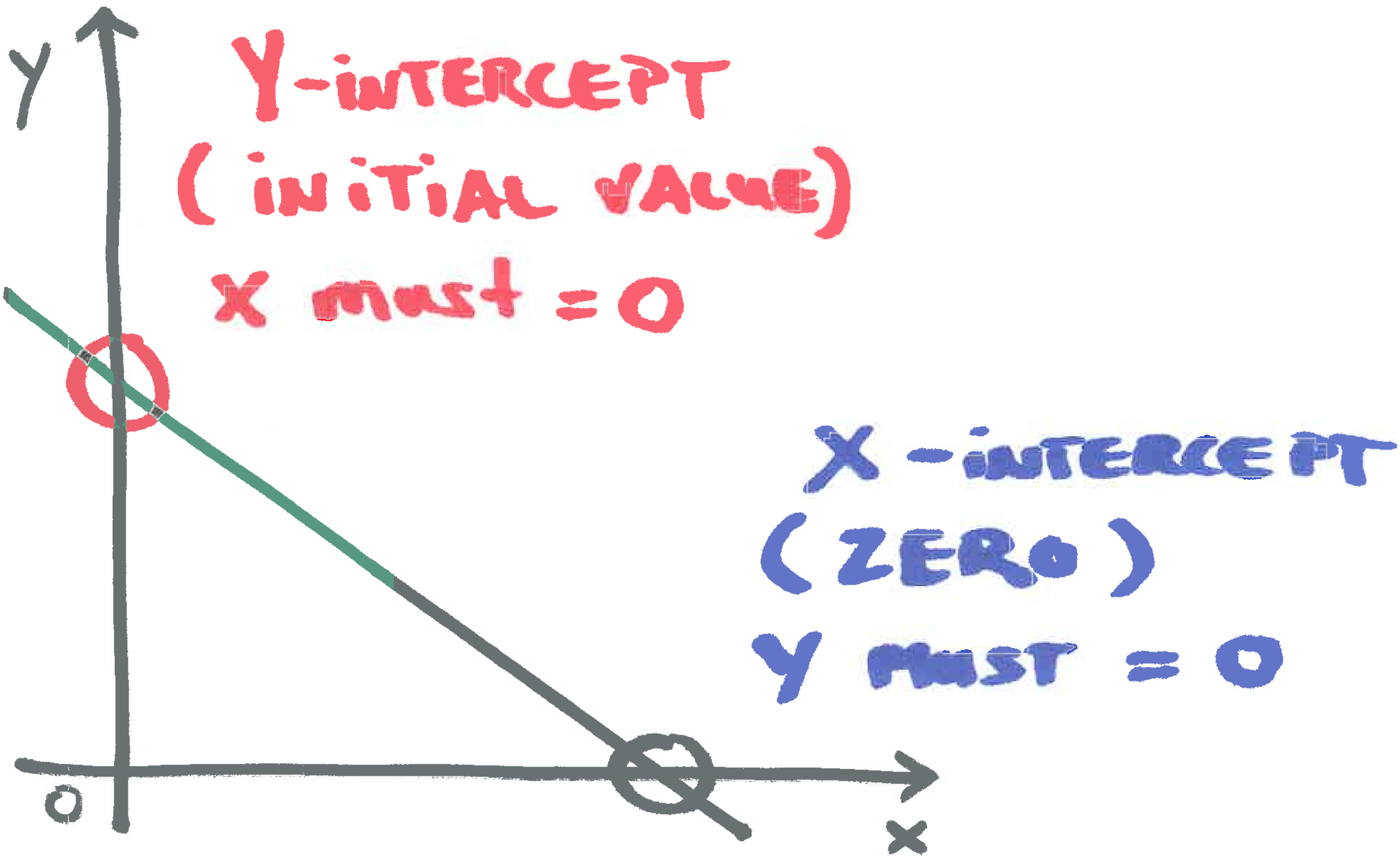
(1) WRITE $y = a \cdot x + b$

(2) FIND SLOPE (a)

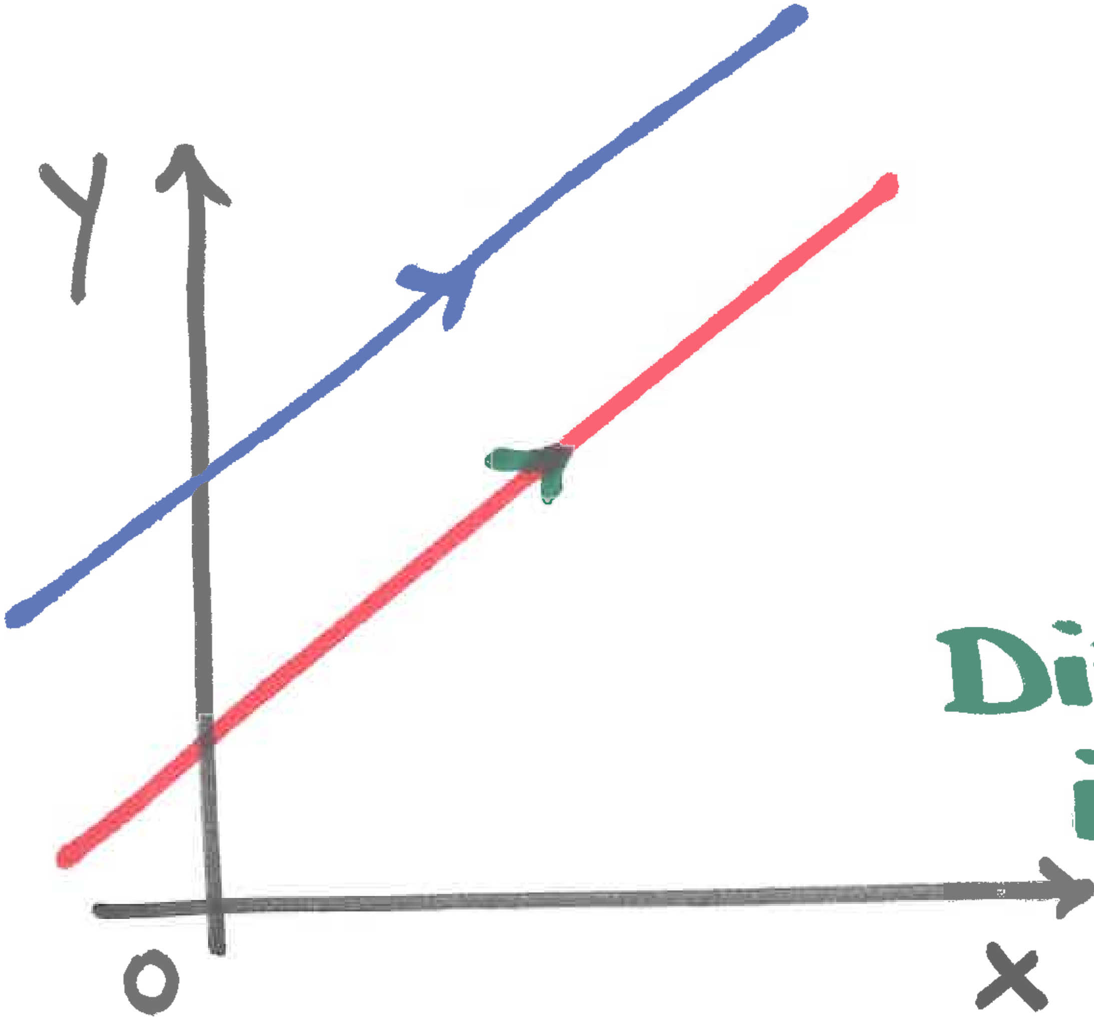
(3) PLUG IN COORDINATES

(4) SOLVE FOR INITIAL VALUE (b)

X AND Y INTERCEPTS



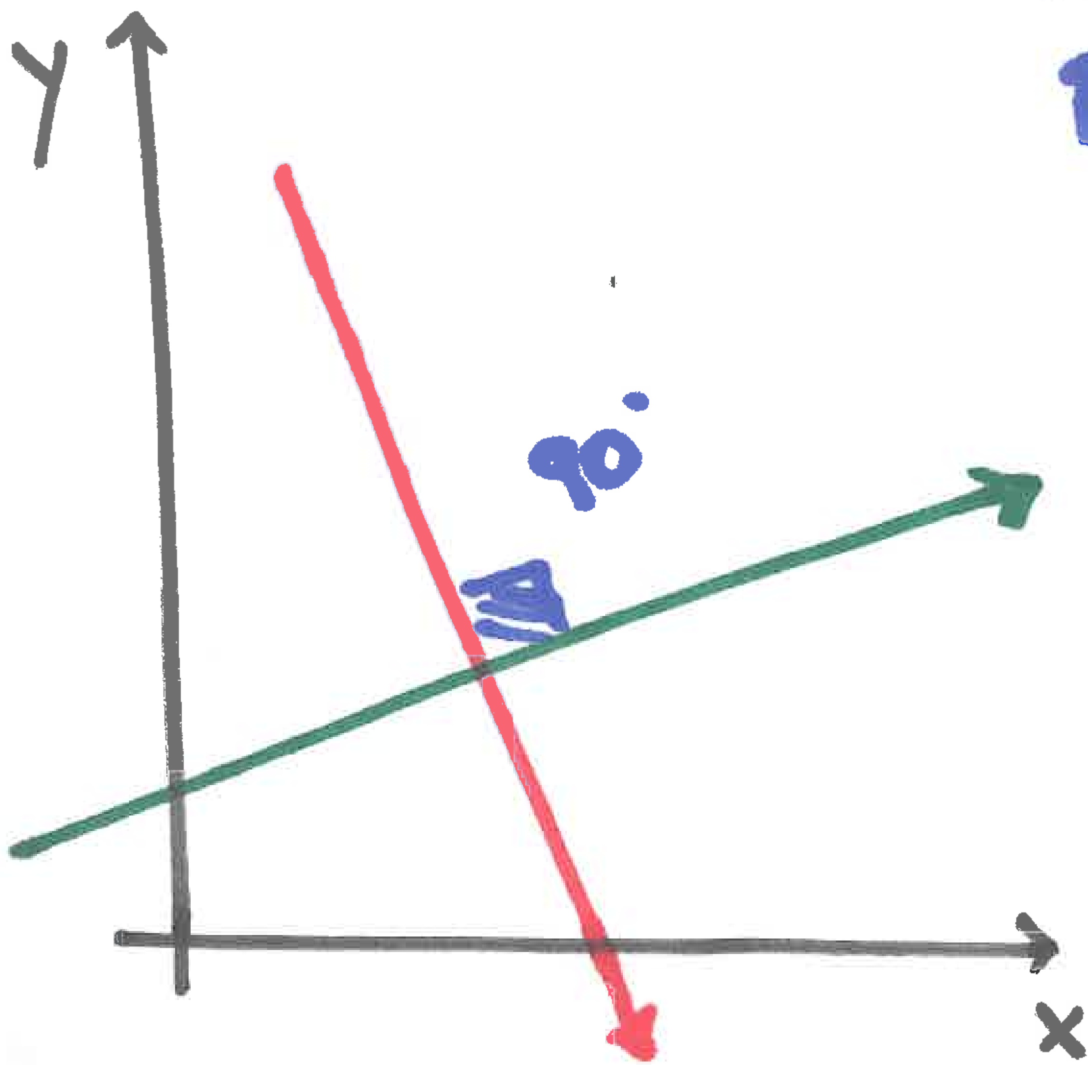
PARALLEL LINES



SAME SLOPE

DIFFERENT INITIAL VALUES.

PERPENDICULAR LINES

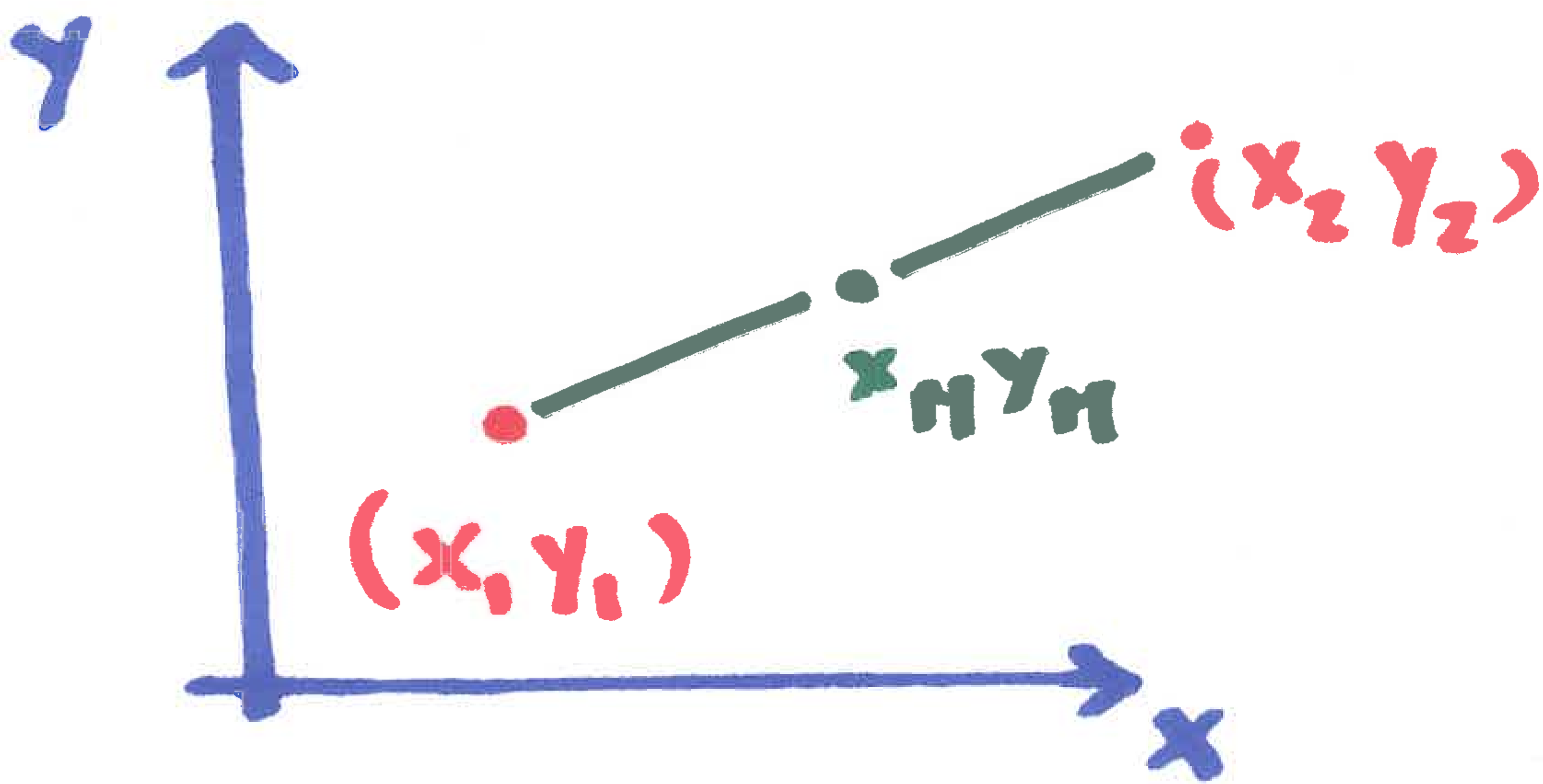


NEGATIVE RECIPROCAL

$$\frac{a}{b} \rightarrow \frac{-b}{a}$$

MIDPOINT

$$x_M, y_M = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$



DIVISION POINT

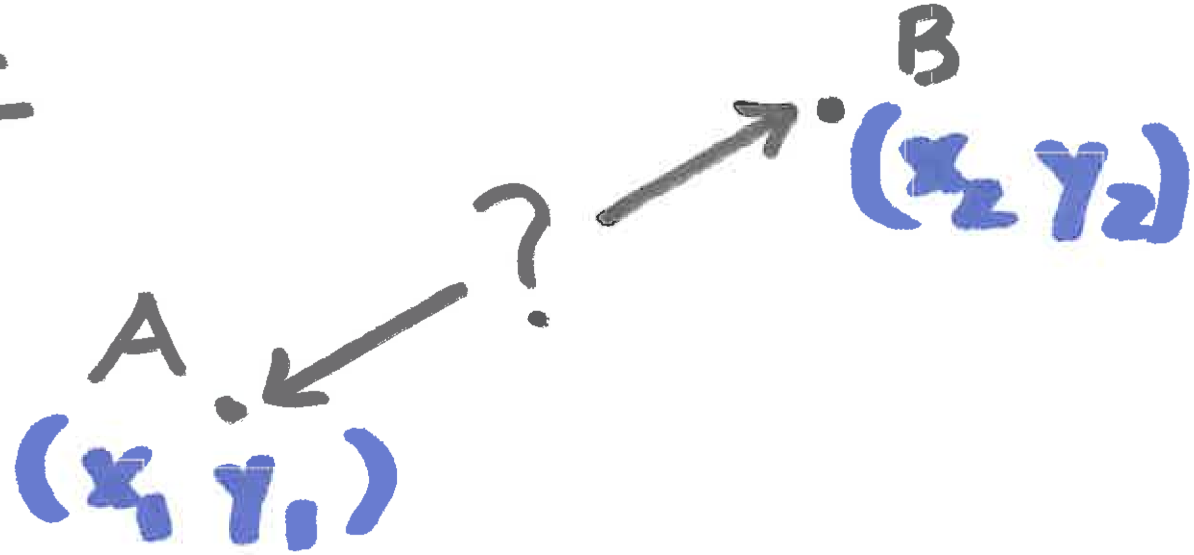
$$(x_D, y_D) =$$

$$= \left(x_1 + \frac{a}{b}(x_2 - x_1),\ y_1 + \frac{a}{b}(y_2 - y_1) \right)$$

RATIO $\longrightarrow$ FRACTION

$$3:2 \longrightarrow \frac{3}{3+2} = \frac{3}{5}$$

DISTANCE



$$dist = \sqrt{(y_2 - y_1)^2 + (x_2 - x_1)^2}$$

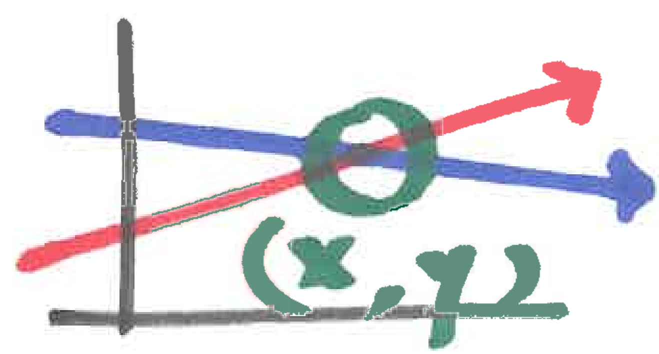
INTERSECTING LINES
(SYSTEMS OF EQUATIONS)

(1) PUT IN $Y = a \cdot x + b$

(2) MAKE EQs. = TO EACH OTHER

(3) SOLVE FOR 'X'

(4) PLUG 'X' BACK IN TO FIND 'Y'



INEQUALITIES

LINEAR INEQUALITIES

$$y < a \cdot x + b$$

$<$ DOTTED LINE, SHADE BELOW

$\leq$ SOLID LINE, SHADE BELOW

$>$ DOTTED LINE, SHADE ABOVE

$\geq$ SOLID LINE, SHADE ABOVE

CHECKING SOLUTIONS TO INEQUALITIES

(1) WRITE THE INEQUALITY

(2) PLUG THE POINT (X,Y) IN

(3) SOLVE AND SEE IF IT WORKS.

POINT (3,2)
x y

$y < 3x + 6$

$y < 3x + 6$

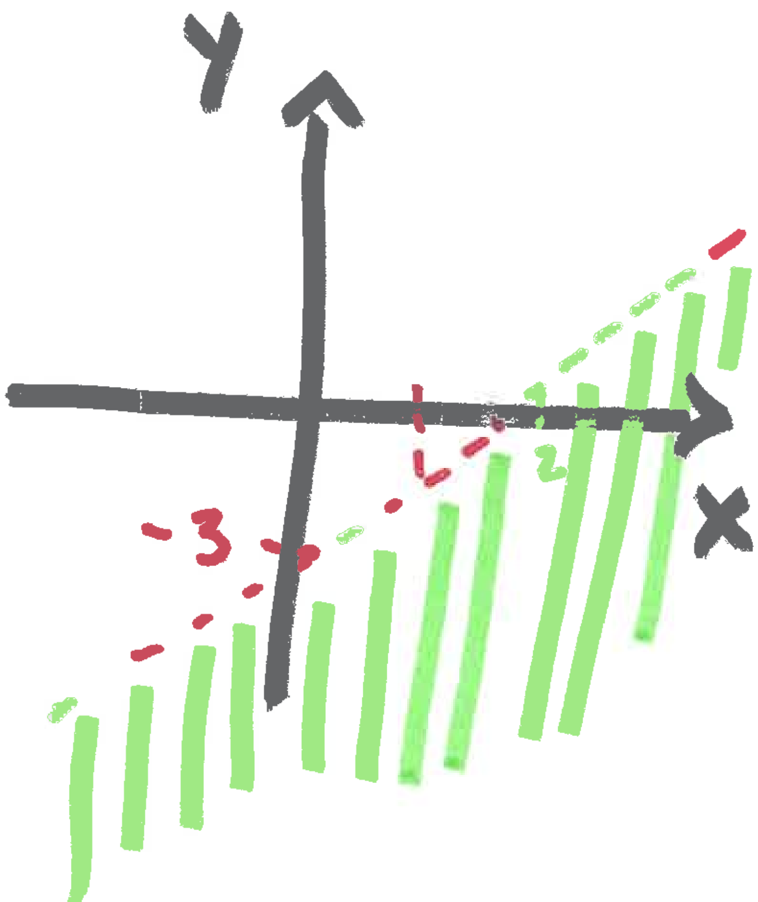
$2 < 3(3) + 6$

$2 < 9 + 6$

$2 < 15$ ✓

SOLVING INEQUALITIES

MULTIPLY OR DIVIDE BY A NEGATIVE, SWITCH THE SIGN.



$$4x - 2y > 6$$
$$-4x \qquad\qquad -4x$$

$$\frac{-2y}{-2} > \frac{-4x}{-2} + \frac{6}{-2}$$

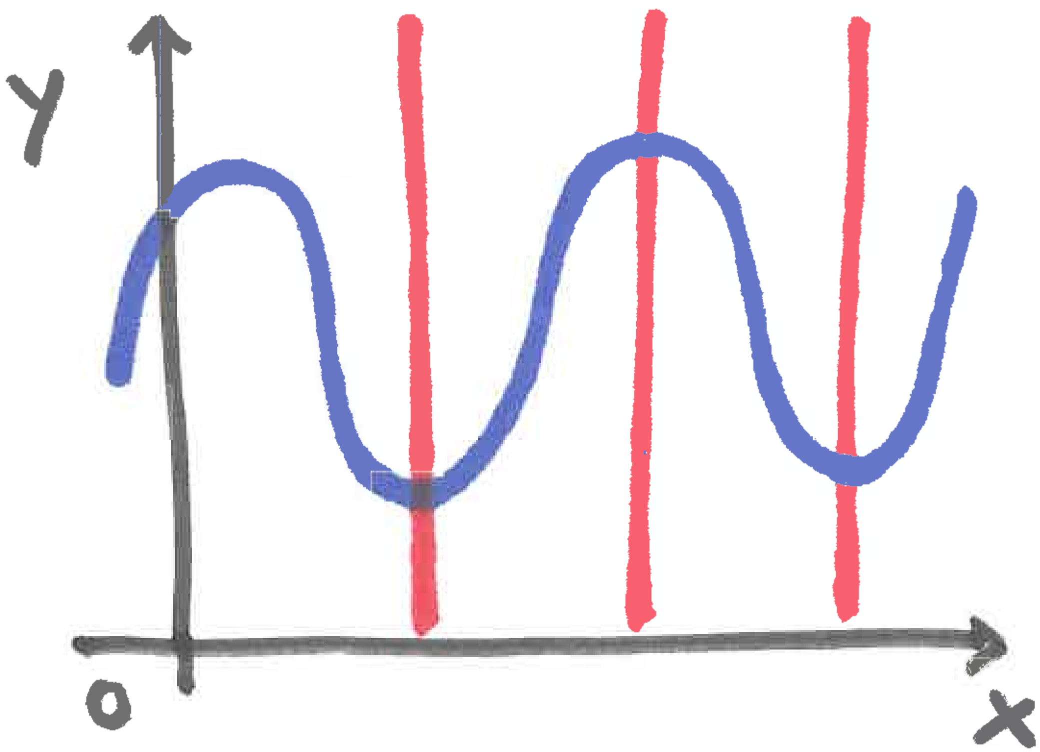
$$y < 2x - 3$$

PROPERTIES OF FUNCTIONS

WHAT IS A FUNCTION?

EVERY X ONLY GIVES ONE Y

PASS THE VLT.



VERTICAL LINE ONLY TOUCHES ONCE !

FUNCTIONS

DOMAIN → 'X' VALUES

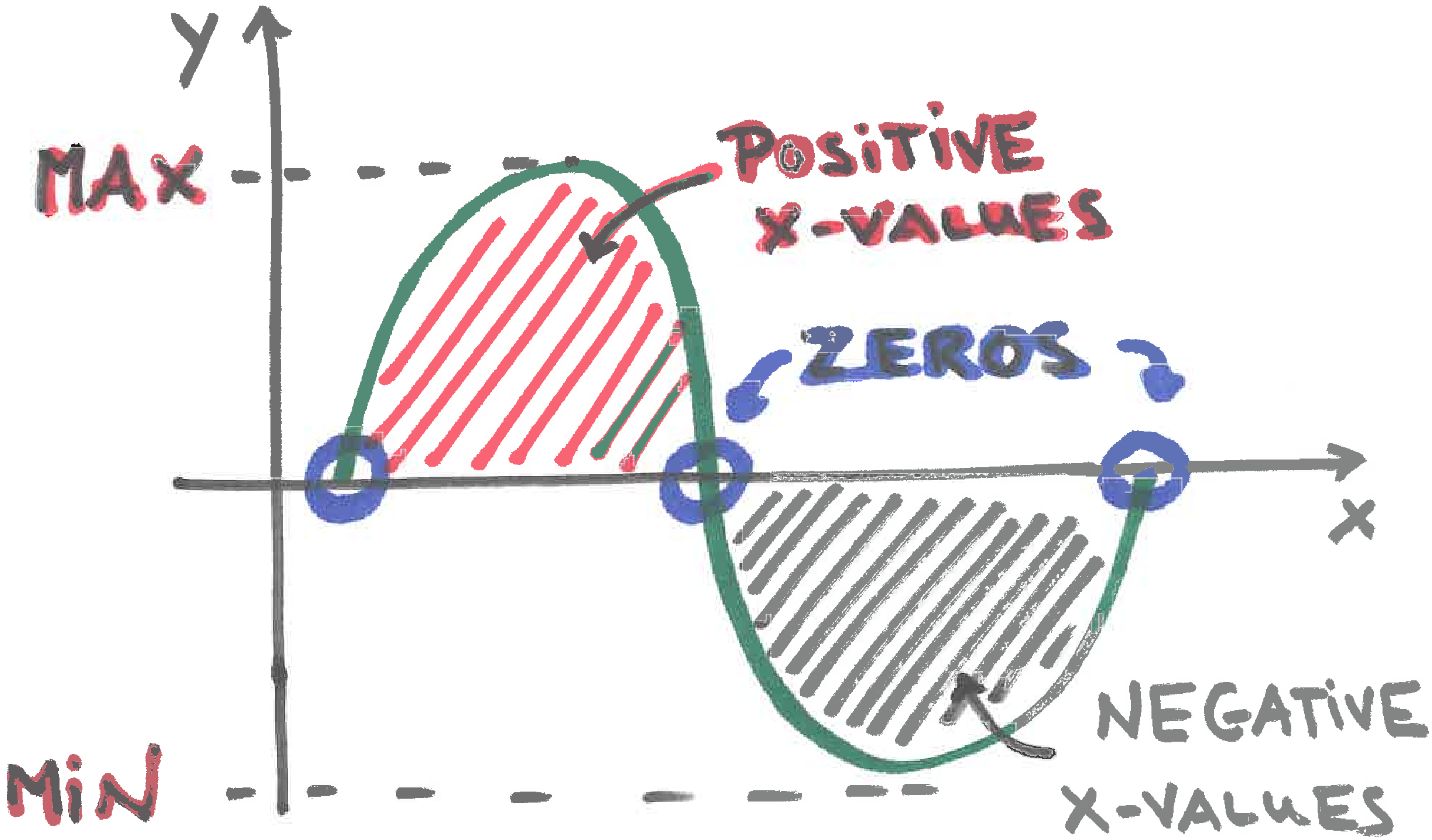
RANGE → 'Y' VALUES

INCREASING → AS 'X' ↑, 'Y' ↑

CONSTANT → AS 'X' ↑, 'Y' STAYS SAME

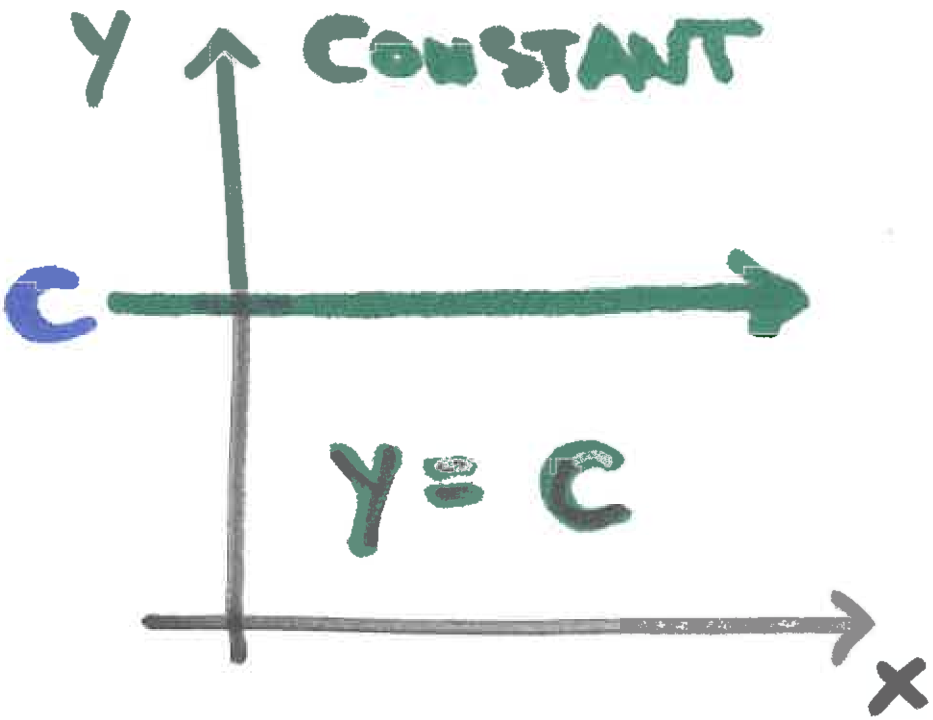
DECREASING → AS 'X' ↑, 'Y' ↓

FUNCTIONS (CONT.)

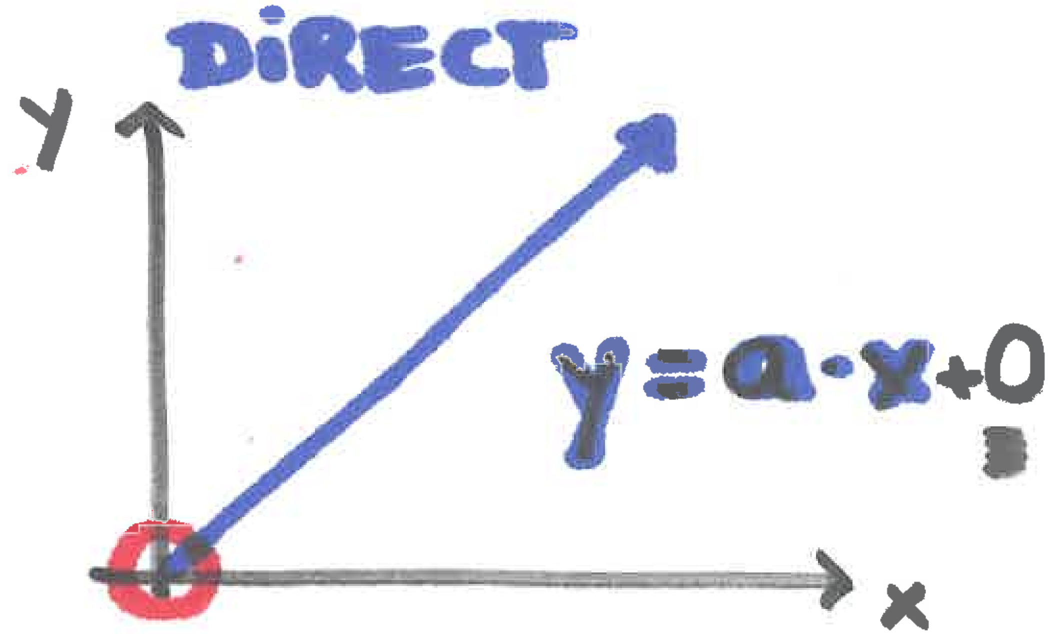


FUNCTIONS

ZERO AND FIRST DEGREE FUNCTIONS.

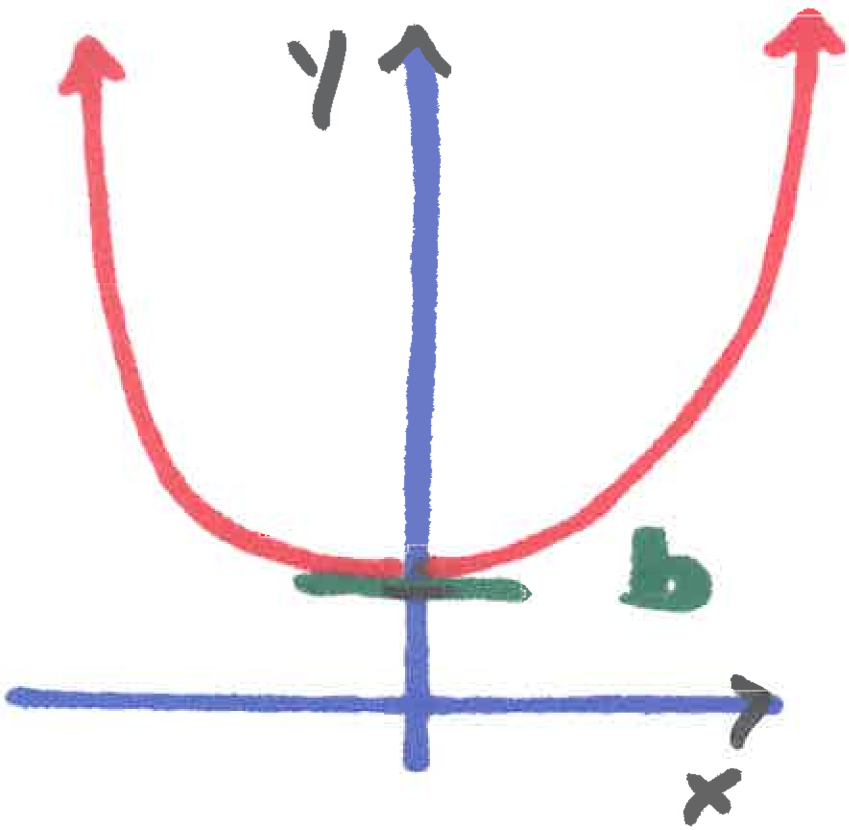


C = a constant number



2^{nd} DEGREE FUNCTION

(QUADRATIC)



$$y = a \cdot x^2 + b$$

WORKING BACKWARD

$$x = \pm\sqrt{\frac{y-b}{a}}$$

Exponential Growth

$$y = a \cdot c^x$$

a = initial #

c = growth rate

Backwards

$$a = \frac{y}{c^x}$$

Growing

10% → 1.10

15% → 1.75

20% → 1.20

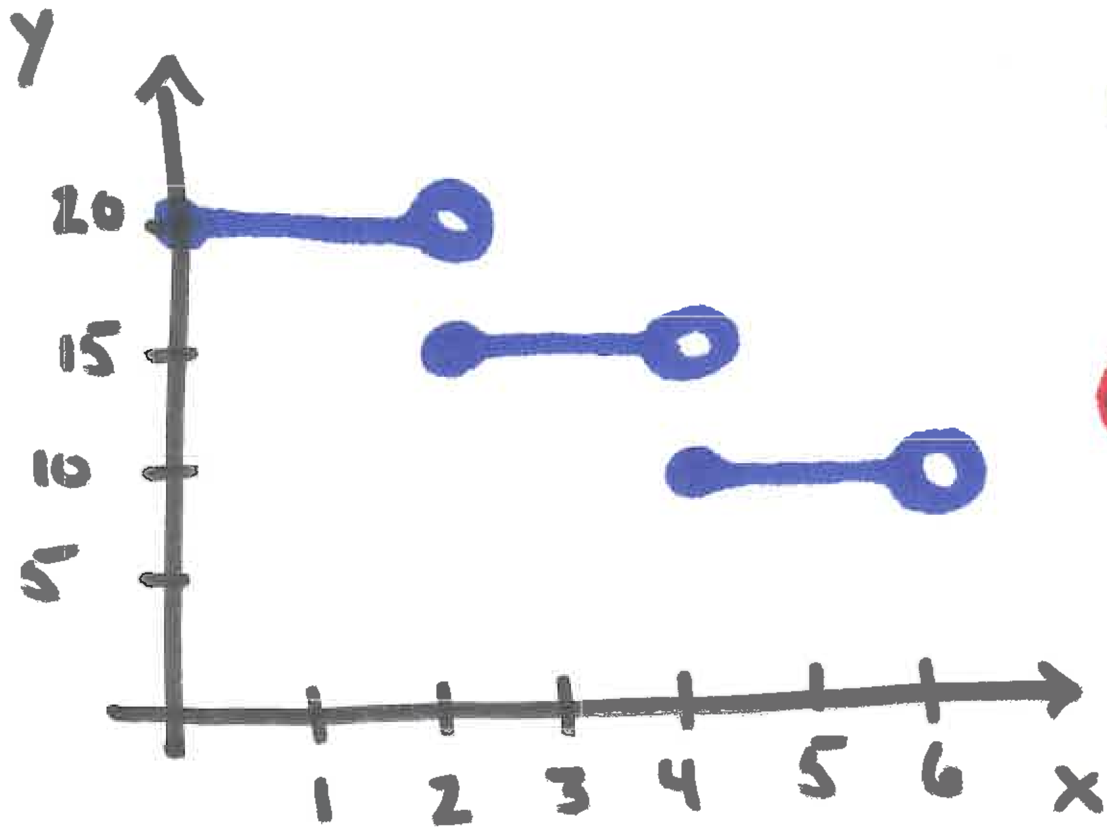
Losing.

10% → 0.90

15% → 0.85

20% → 0.80

STEP FUNCTION



PASS RIGHT THROUGH

STOP AND READ THE Y-VALUE

f(2) = 15

f(3) = 15

f(4) = 10

PERIODIC FUNCTION

(1) FIND THE PERIOD (TIME FOR A FULL CYCLE)

(2) HOW MANY FULL CYCLES.

(3) HOW MUCH TIME DO THE FULL CYCLES TAKE UP.

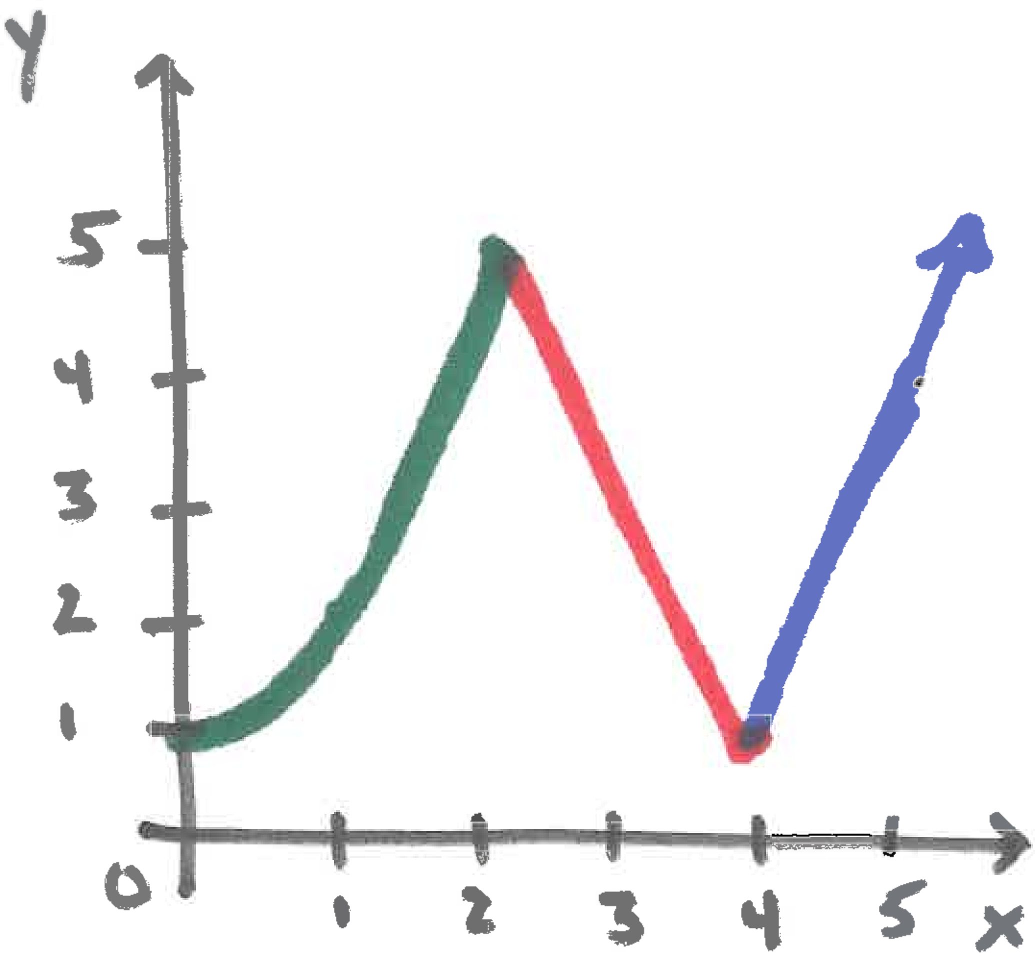
(4) HOW MUCH TIME IS LEFT OVER.

(5) FIND THE RULE USING 2 POINTS ON THE GRAPH

(6) PLUG IN THE TIME (X) AND FIND THE ANSWER (Y).

PIECE-WISE FUNCTION

DIFFERENT FUNCTIONS AT DIFFERENT POINTS ALONG THE DOMAIN.



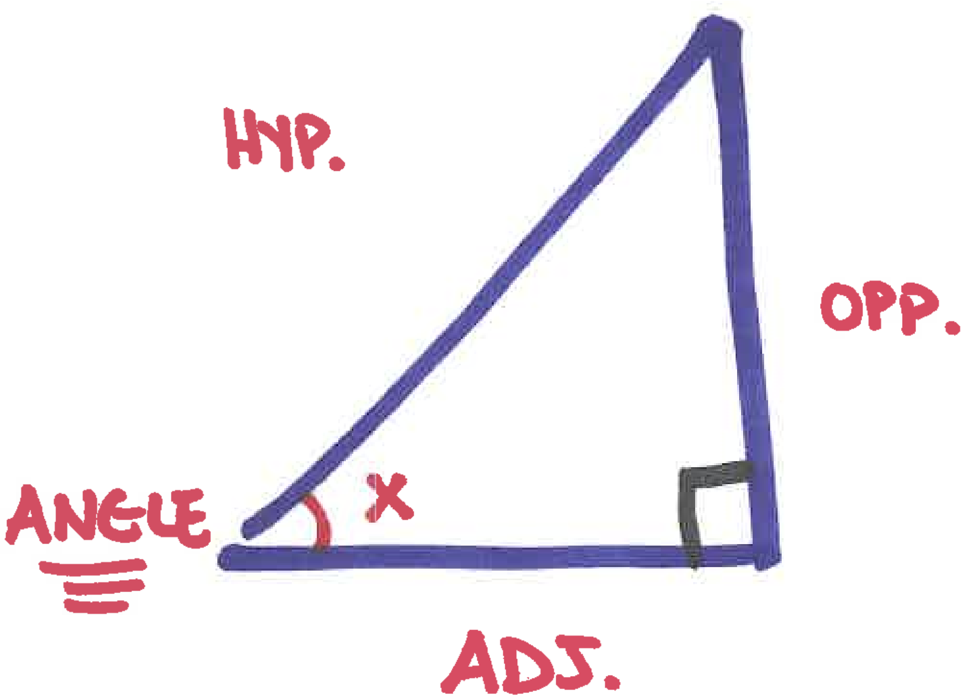
$$f(x) = \begin{cases} x^2+1, & 0 \le x \le 2 \\ -2x+9, & 2 \le x \le 4 \\ 3x-11, & 4 \le x \end{cases}$$

TRIGONOMETRY

SOH - CAH - TOA

(FOR RIGHT △s)

θ means angle ($\measuredangle$)

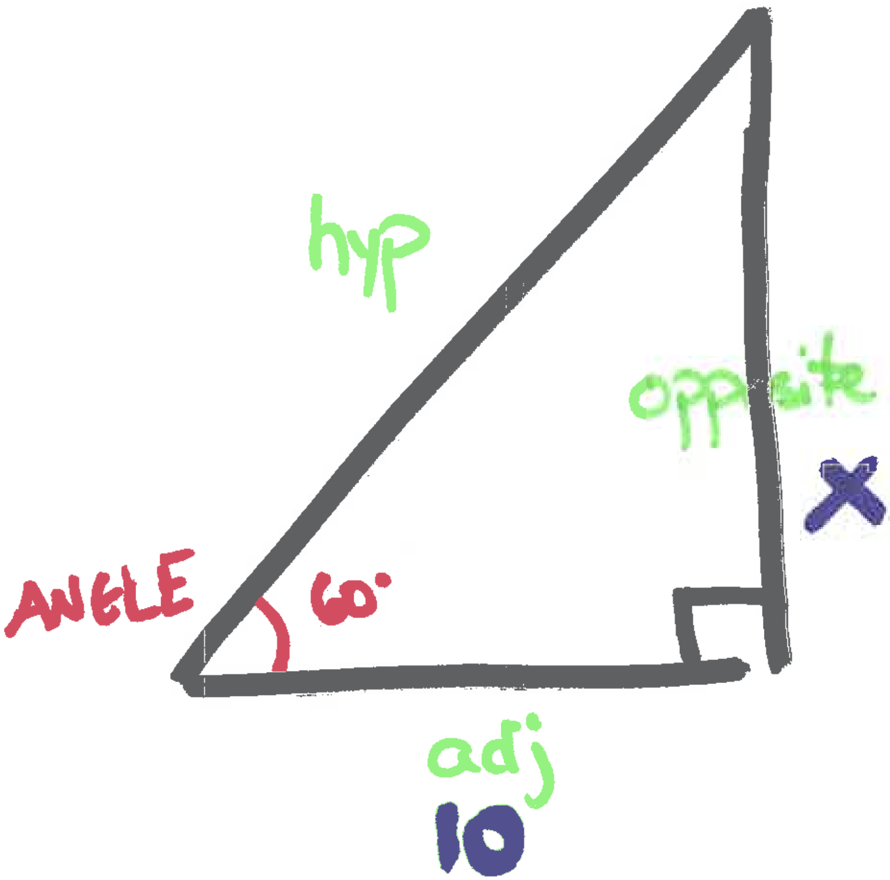


$$\sin\theta = \frac{\text{OPP}}{\text{HYP}}$$

$$\cos\theta = \frac{\text{ADJ}}{\text{HYP}}$$

$$\tan\theta = \frac{\text{OPP}}{\text{ADJ.}}$$

SOH CAH TOA (SIDE)



(1) IDENTIFY ANGLE

(2) LABEL SIDES

(3) CHOOSE RIGHT SOH CAH TOA

(4) SOLVE FOR MISSING SIDE.

$$\boxed{\tan\theta = \frac{opp}{adj}}$$

$$\frac{\tan 60}{1} = \frac{x}{10}$$

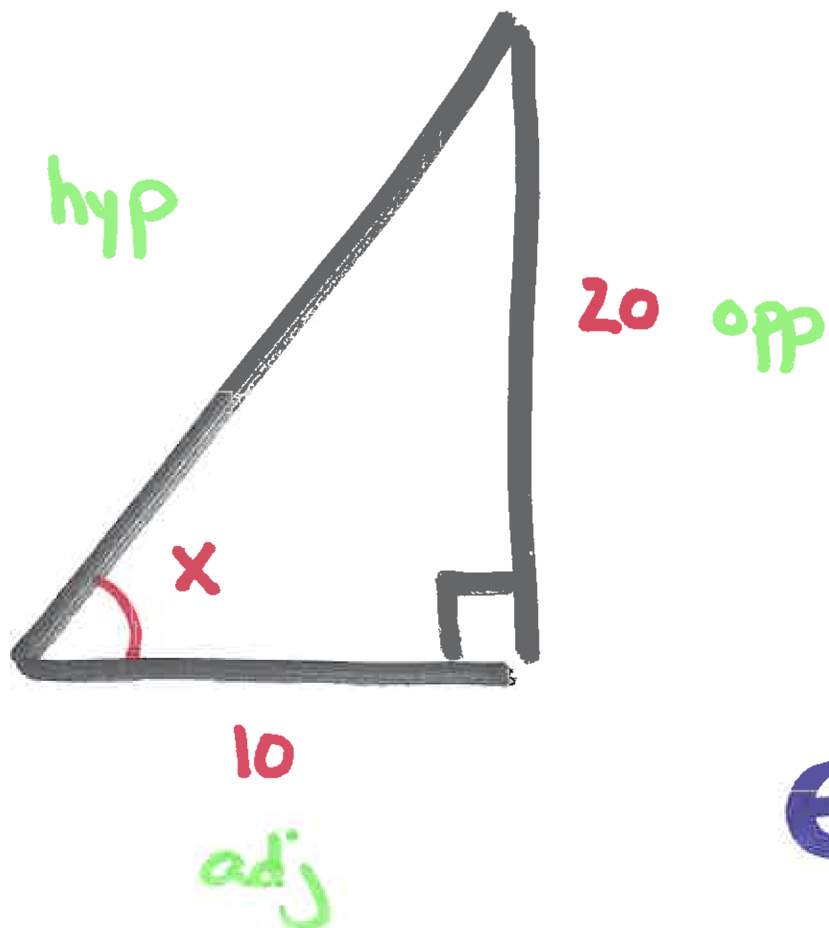
$$\boxed{x = 10 \cdot (\tan 60)}$$

HAVE: ADJ

WANT: OPP

TOA

SOH - CAH - TOA (ANGLE)



HAVE: OPP
ADJ

USE TOA

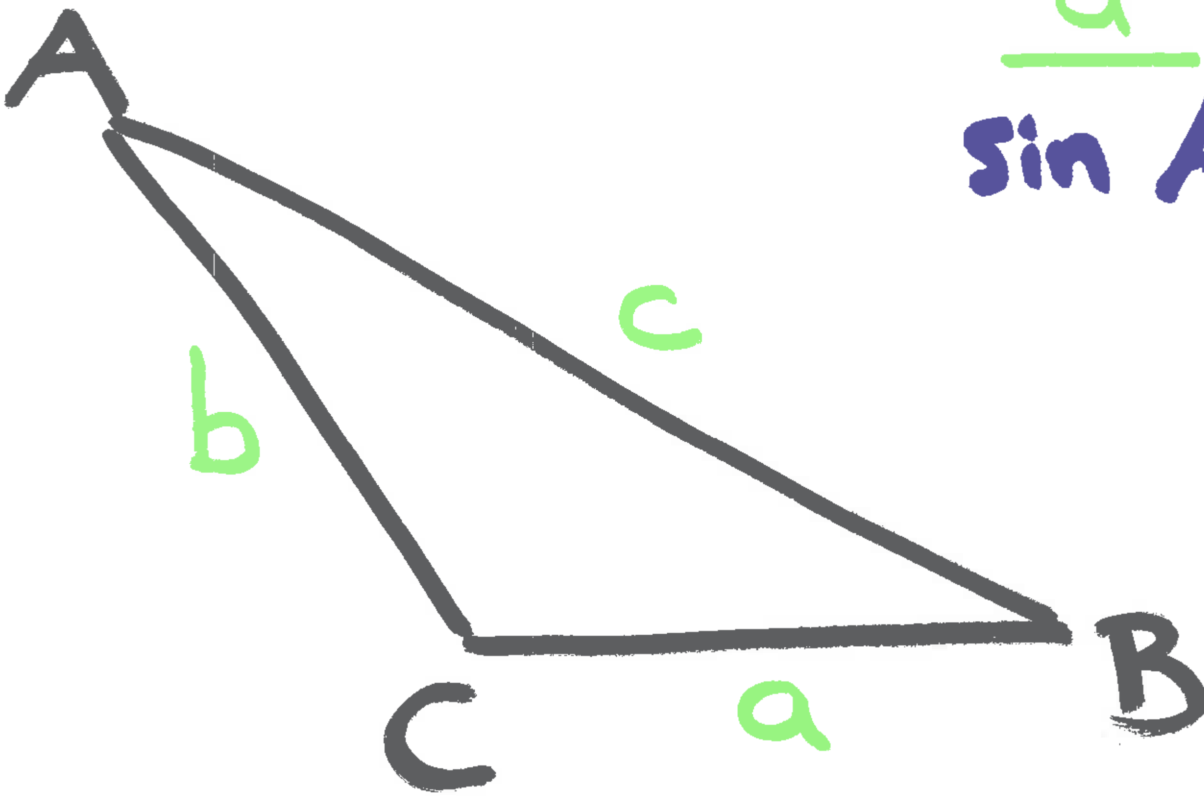
$$\tan\theta = \frac{opp}{adj}$$

$$\theta = \tan^{-1}\left(\frac{opp}{adj}\right)$$

Ex:

$$\boxed{x = \tan^{-1}\left(\frac{20}{10}\right) = 63.4^\circ}$$

SINE LAW



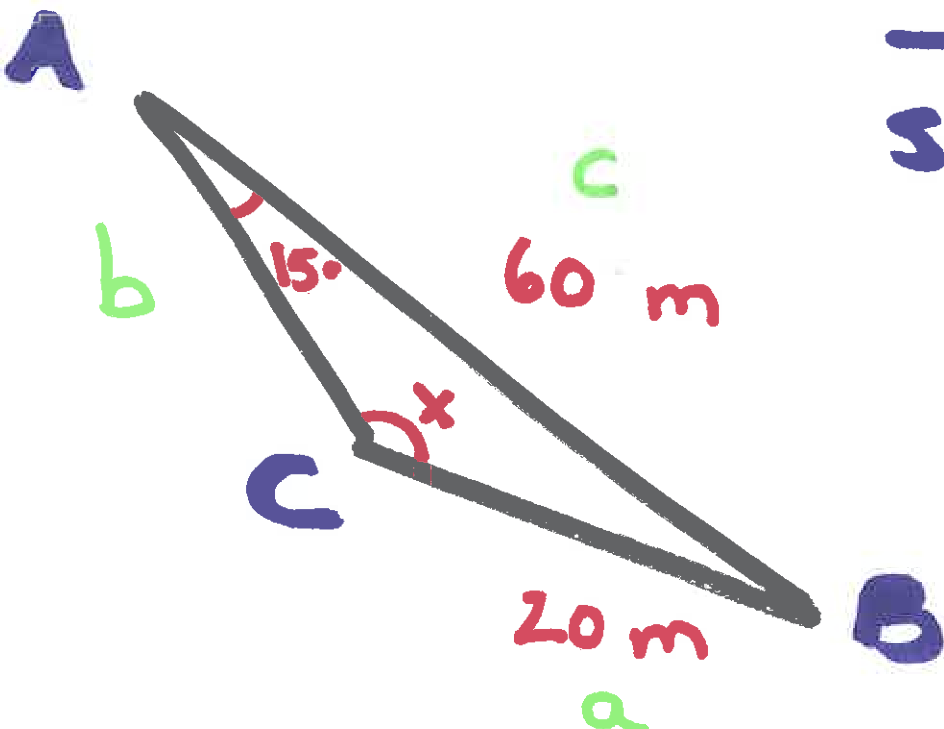
$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

NEED 3 of 4...

* OBTUSE ANGLES =

180° − ACUTE ANGLE.

SINE LAW



$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$\frac{60}{\sin x} = \frac{20}{\sin 15°}$$

$$x = \sin^{-1}\left(\frac{60 \cdot (\sin 15)}{20}\right)$$

$$x = 50.94°$$

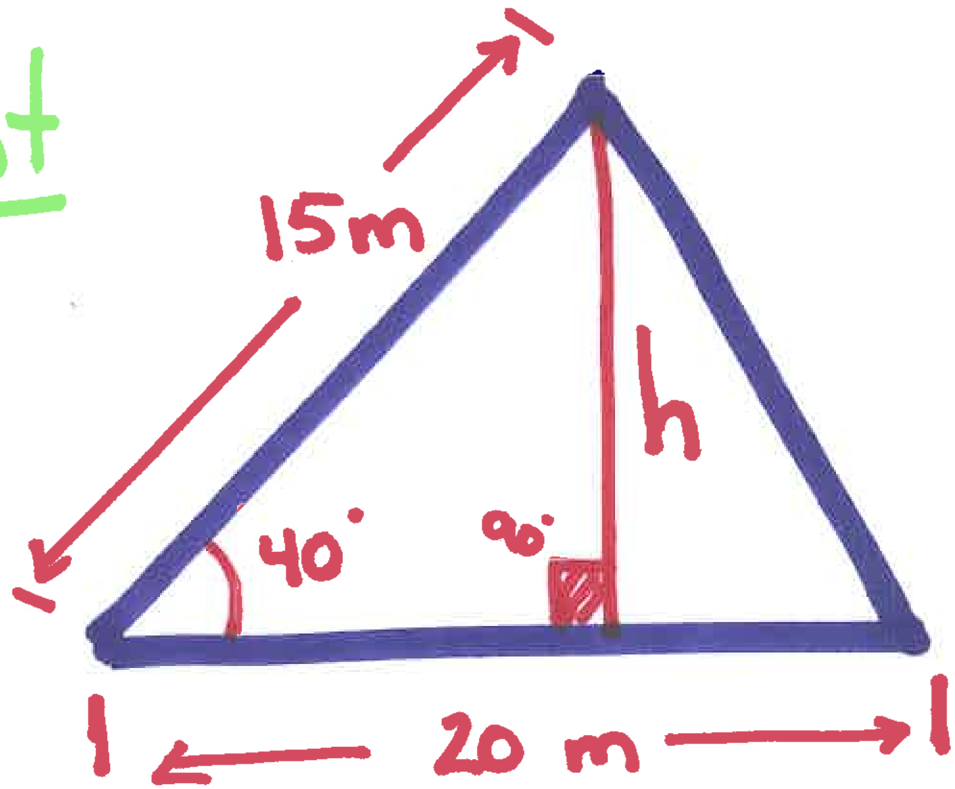
REMEMBER:

OBTUSE = 180 − ACUTE

OBTUSE = 180 − 50.94 = 129.06°

AREA of a Δ

$$A = \frac{\text{base} \cdot \text{height}}{2}$$



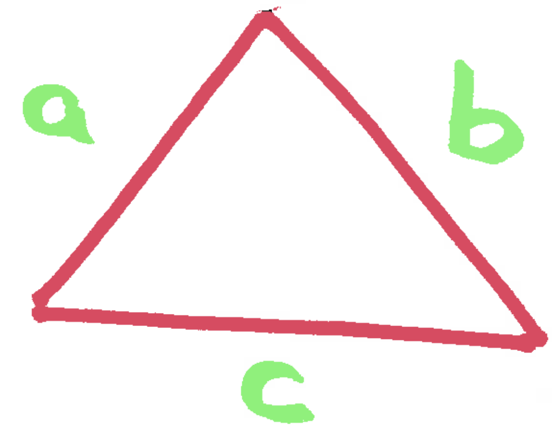
$$\frac{h}{\sin 40^\circ} = \frac{15}{\sin 90^\circ}$$

$$h = \frac{15 \cdot (\sin 40)}{1}$$

$$A = \frac{(20) \cdot (15 \cdot \sin 40)}{2}$$

$$\boxed{A = 96.418 \text{ m}^2}$$

HERO'S LAW



PERIMETER = a + b + c

$$p = \frac{\text{PERIMETER}}{2} = \frac{a+b+c}{2}$$

$$\text{AREA} = \sqrt{p \cdot (p-a) \cdot (p-b) \cdot (p-c)}$$

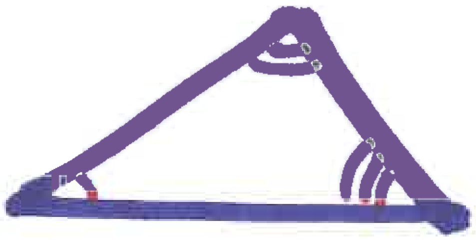
REMEMBER: p is half the PERIMETER.

TRIANGLES

ISOMETRY

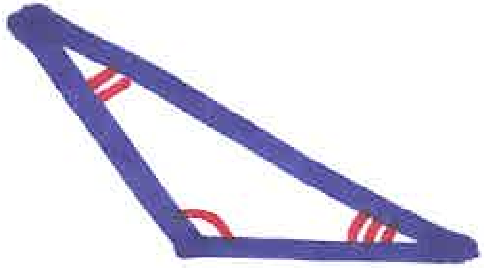
SIMILITUDE

TRIANGLES



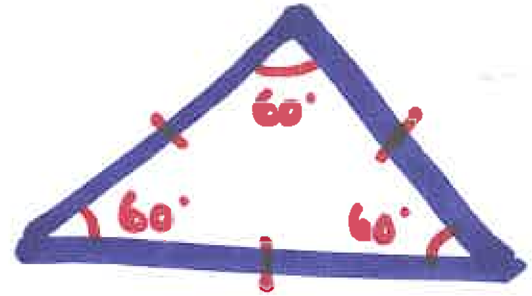
ACUTE

(all $< 90^\circ$)



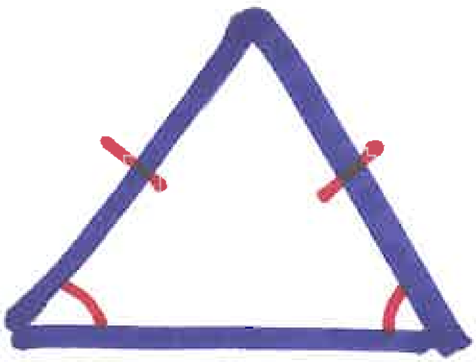
OBTUSE

(one $> 90^\circ$)



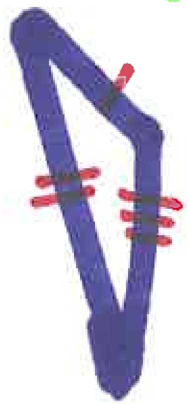
EQUILATERAL

(all equal)



ISOCELES

(2 sides, angles congruent)

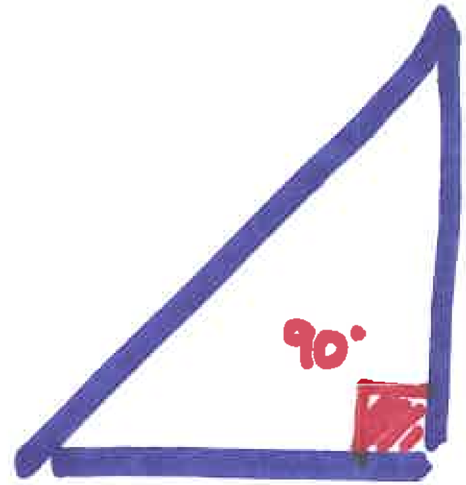


SCALENE

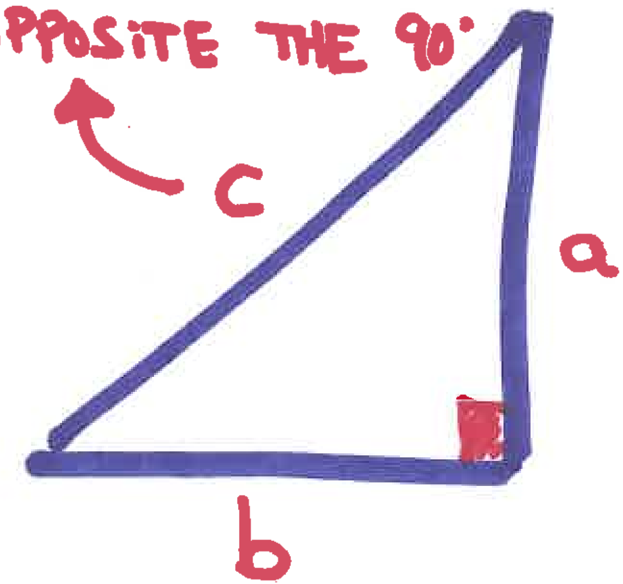
(all different).

PYTHAGORAS

ONLY FOR RIGHT △s.



ALWAYS LONGEST
OPPOSITE THE 90°



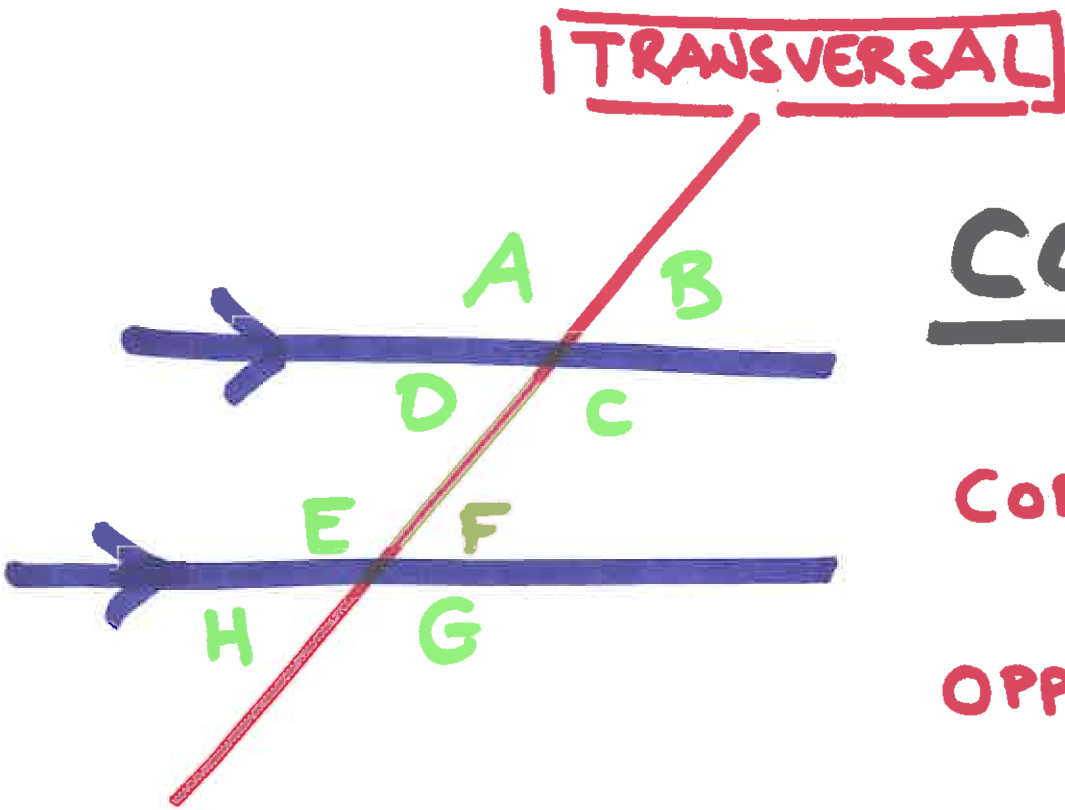
$$\boxed{a^2 + b^2 = c^2}$$

$$c = \sqrt{a^2 + b^2}$$

$$a = \sqrt{c^2 - b^2}$$

$$b = \sqrt{c^2 - a^2}$$

// LINES AND TRANSVERSALS



CONGRUENT (same)

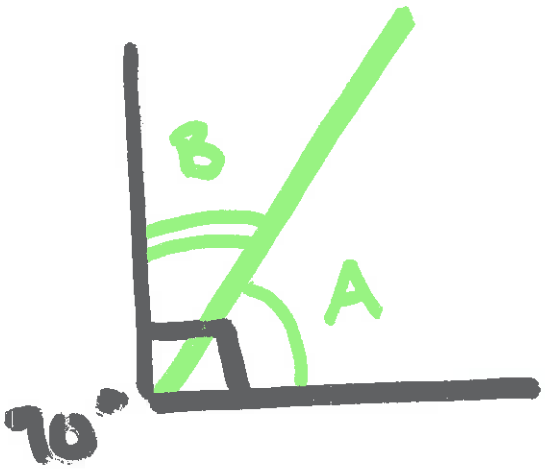
CORRESPONDING (A, E)

OPPOSITE (A, C)

ALT. INT (D, F)

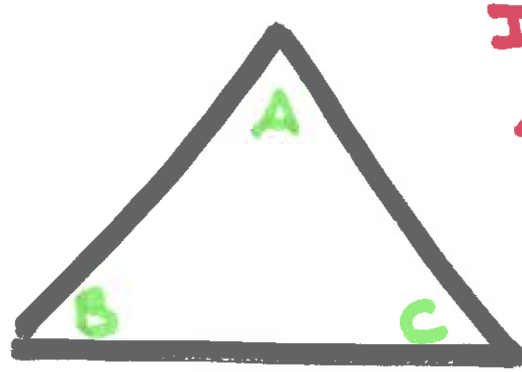
ALT EXT. (A, G)

ANGLE RELATIONSHIPS



COMPLEMENTARY

ADD UP TO 90°



INSIDE A Δ

ADD UP TO 180°

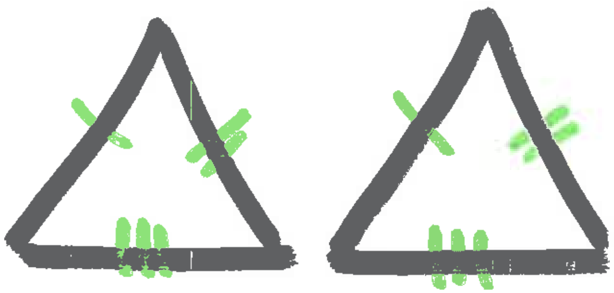
SUPPLEMENTARY

ADD UP TO 180°

ISOMETRY

(CONGRUENCY)

SSS

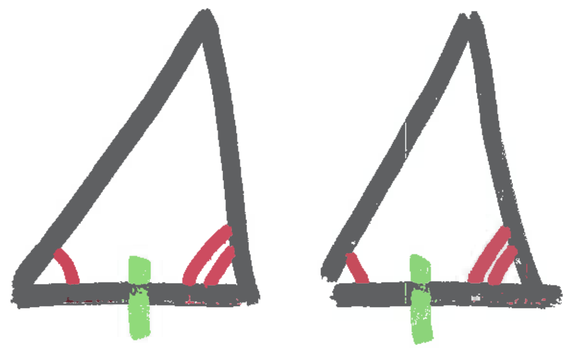


CORRESPONDING SIDES ARE CONGRUENT

SAS

ANGLE IS CONGRUENT AND BETWEEN CORRESPONDING CONGRUENT SIDES.

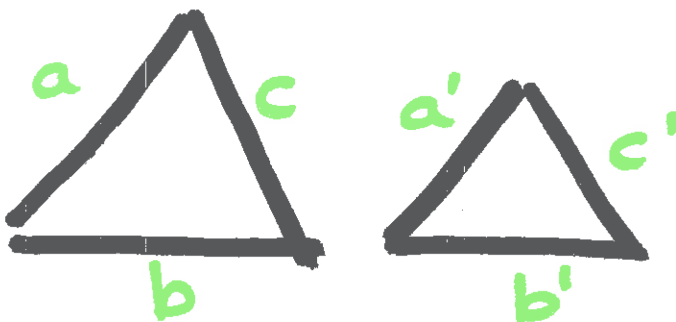
ASA



CORRESPONDING SIDE IS CONGRUENT AND BETWEEN CORRESPONDING CONGRUENT ANGLES.

SIMILARITY

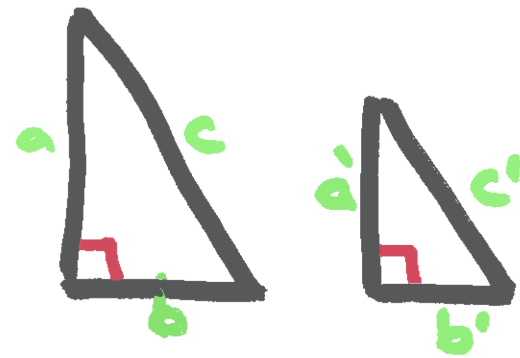
SSS



Corresponding sides are proportional

$$\frac{\text{long}}{\text{long}} = \frac{\text{med}}{\text{med}} = \frac{\text{short}}{\text{short}}$$

SAS

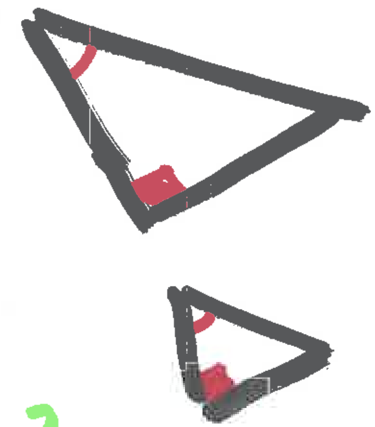


Corresponding sides are proportional.

AND

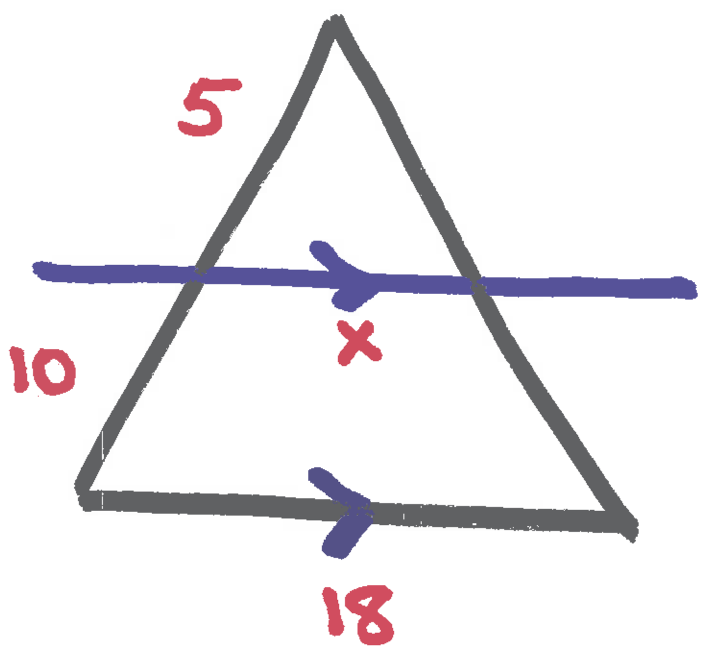
ANGLE BETWEEN SIDES is CONGRUENT.

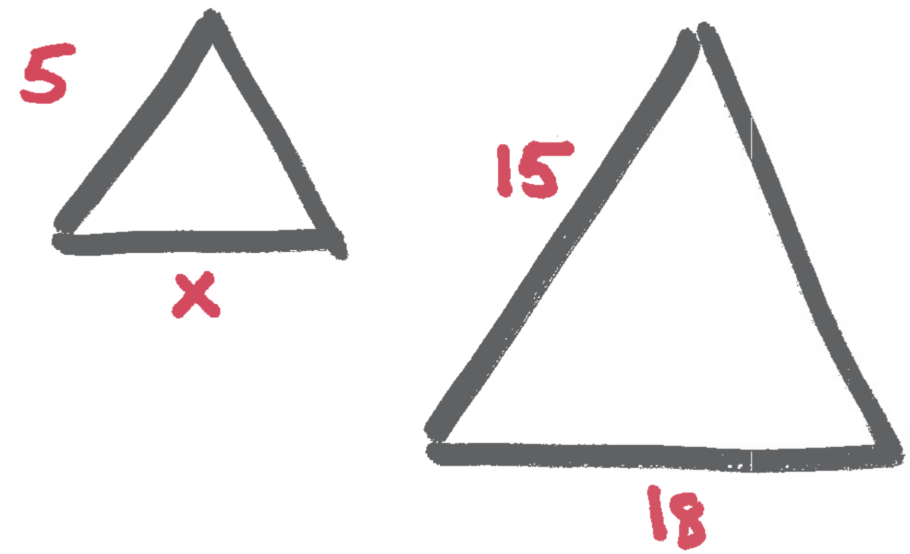
AA



2 CORRESPONDING ANGLES ARE CONGRUENT.

FINDING UNKNOWNS IN SIMILAR FIGURES



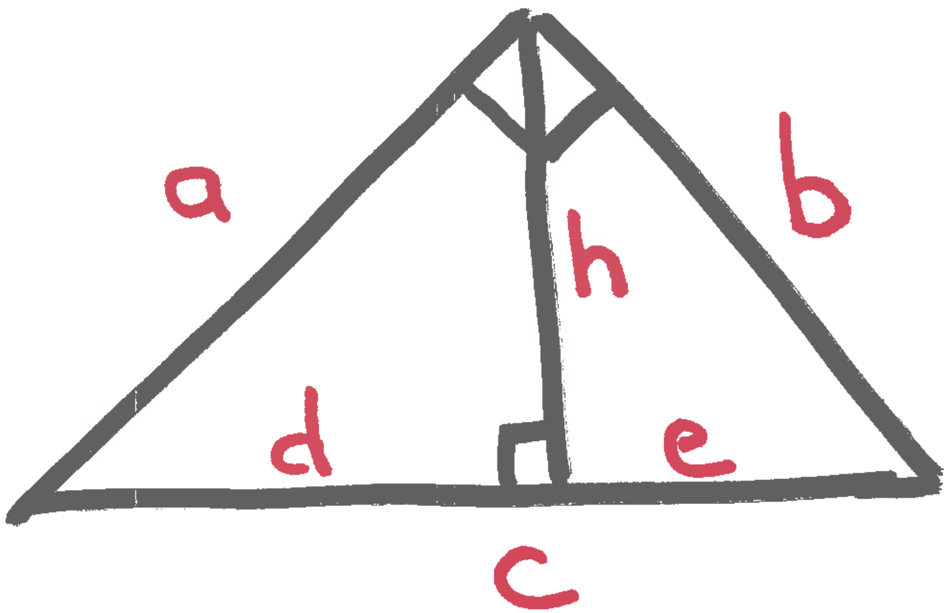


$$\frac{15}{5} = \frac{18}{x}$$

$$x = \frac{18 \cdot 5}{15} = 6$$

$$\boxed{x = 6}$$

METRIC RELATIONS



$$a^2 = c \cdot d$$

$$b^2 = c \cdot e$$

$$h^2 = d \cdot e$$

$$a \cdot b = c \cdot h$$

$$c = d + e$$

a=
b=
c=
d=
e=
h=

PROBABILITY

PROBABILITY

$\cap \longrightarrow \times$

"AND" MEANS TIMES

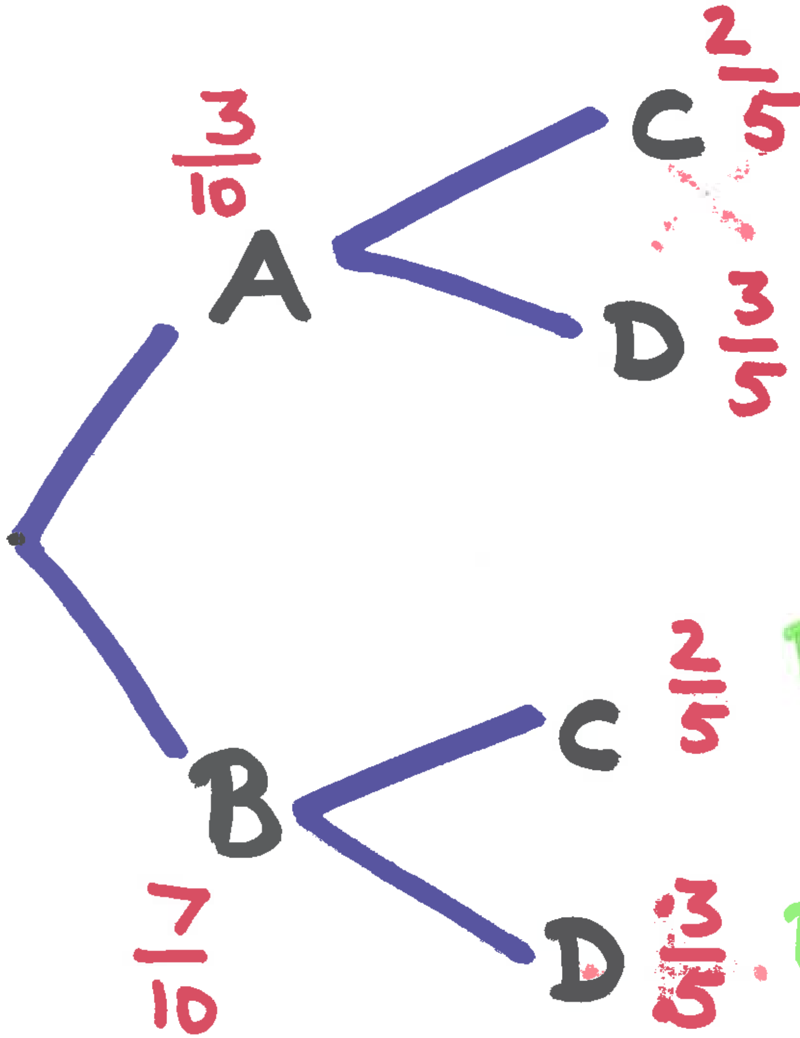
$$P(A \cap B) = P(A) \cdot P(B)$$

$\cup \longrightarrow +$

"OR" MEANS PLUS

$$P(A \cup B) = P(A) + P(B)$$

PROBABILITY



$P(A \cap C) = \frac{3}{10} \cdot \frac{2}{5} = \frac{6}{50} = 12\%$

$P(A \cap D) = \frac{3}{10} \cdot \frac{3}{5} = \frac{9}{50} = 18\%$

$P(B \cap C) = \frac{7}{10} \cdot \frac{2}{5} = \frac{14}{50} = 28\%$

$P(B \cap D) = \frac{7}{10} \cdot \frac{3}{5} = \frac{21}{50} = 42\%$

$P(A \cap C \cup B \cap C) = 12\% + 28\% = 40\%$

PROBABILITY AND ODDS

ODDS $\longleftrightarrow$ PROBABILITY

$$1:3 \longleftrightarrow \frac{1}{4}$$

first # in odds

total of all the odds.

ODDS FOR AND AGAINST.

2:1 ODDS "FOR" means 2 : 1

WAYS TO WIN — WAY TO LOSE

3:4 ODDS "AGAINST" MEANS 3 : 4

WAYS TO LOSE — WAYS TO WIN.

BETTING

BET 50$ ON A 2:1 UNDERDOG.

2:1 ⇒ 1:2

lose win — win lose.

→ $\frac{1}{3}$ CHANCE TO WIN.

$$\boxed{\frac{1}{3} = \frac{50}{?}}$$

$?= \frac{50 \cdot 3}{1} =$ 150$ collected

− 50 original bet

100$ PROFIT.

MATHEMATICAL EXPECTATION.

$$M = \underbrace{x_1}_{\text{AMOUNT WIN}} \cdot \underbrace{p(x_1)}_{\text{probability of WINNING.}} + \underbrace{x_2}_{\text{AMOUNT LOSE}} \cdot \underbrace{p(x_2)}_{\text{PROBABILITY of LOSING.}}$$

AMOUNT WIN · probability of WINNING. + AMOUNT LOSE · PROBABILITY of LOSING.

FAIR GAMES

FAIR if
$\mu = 0$

TO MAKE A GAME FAIR CHANGE THE AMOUNT \$ WON OR LOST.

if μ = NEGATIVE, PLAYER SHOULD LOSE MORE \$.

μ = POSITIVE, PLAYER SHOULD WIN MORE \$.

STATISTICS

CENTRAL TENDENCY

MEAN ($\bar{X}$) = $\dfrac{\text{SUM OF SCORES}}{\text{NUMBER OF SCORES}}$

MEDIAN (Md) = PUT #s IN ORDER

ODD # → PICK MIDDLE

EVEN # → AVG. of 2 MIDDLE

MODE (Mo) = MOST OFTEN

CAN HAVE MORE THAN 1.

MEAN DEVIATION

(1) FIND THE MEAN ($\bar{X}$)

(2) FIND THE DISTANCE OF EACH SCORE FROM THE MEAN

(3) MAKE EACH DISTANCE POSITIVE

(4) FIND THE MEAN OF THE DISTANCES.

PERCENTILE

ALWAYS ROUND UP!

$$P = \frac{\text{\# SCORES LOWER AND EQUAL}}{\text{TOTAL NUMBER OF SCORES}} \times 100$$

WORKING BACKWARD

$$\text{SCORE} = \frac{\text{PERCENTILE}}{100} \times \text{TOTAL}$$

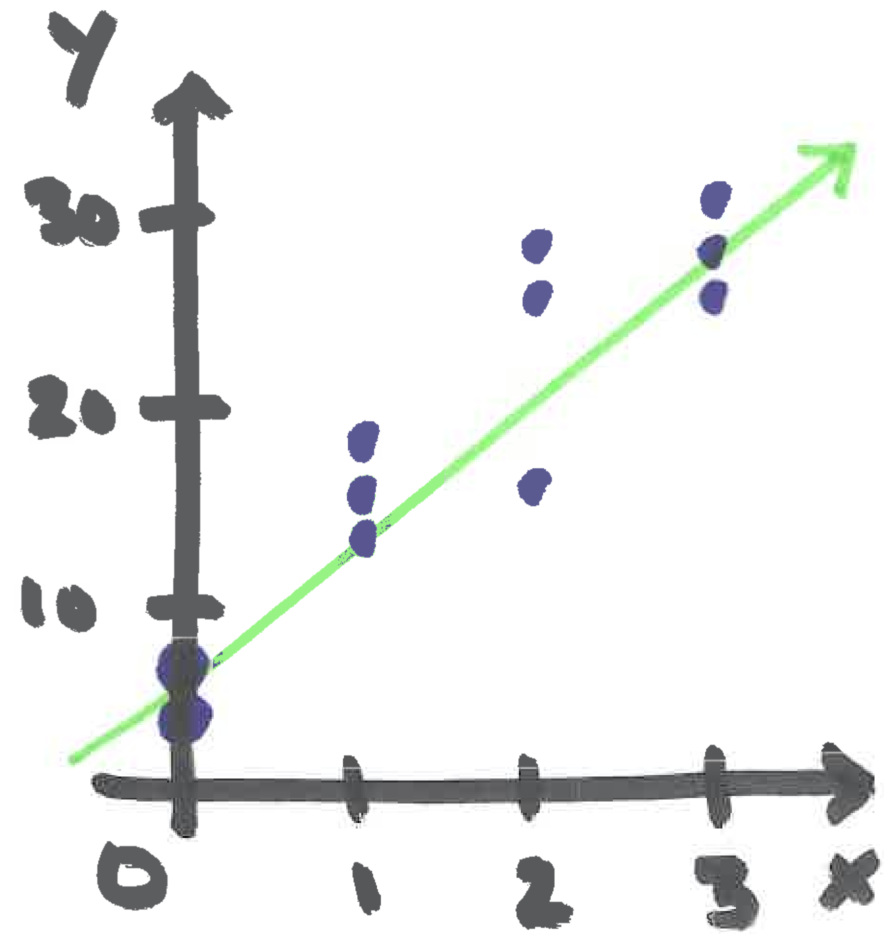
ALWAYS ROUND DOWN!

CONTINGENCY TABLES

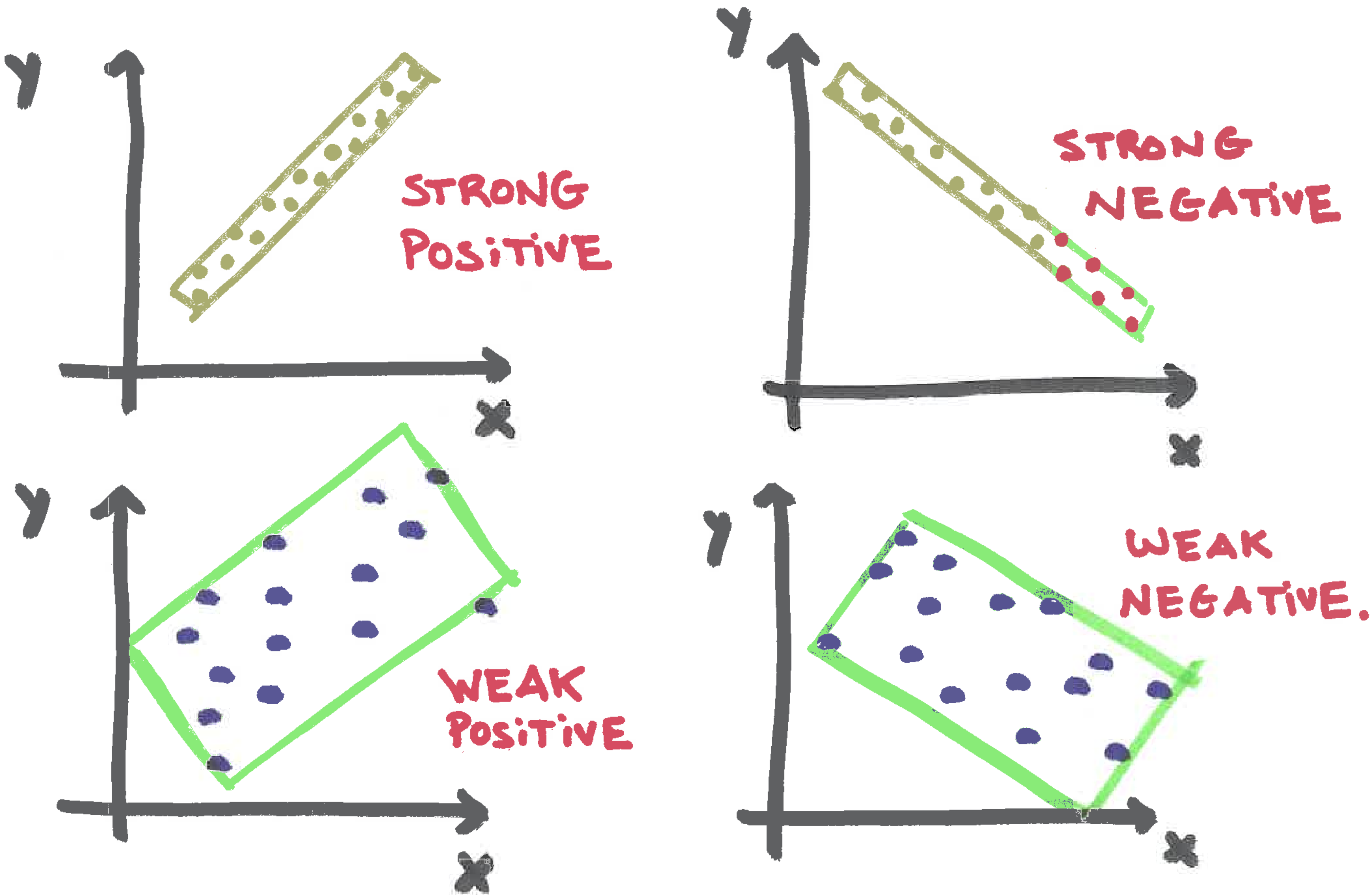
for scatter plots.

x \ y	0-10	10,20	20,30
0	2	0	0
1	0	3	0
2	0	1	2
3	0	0	3

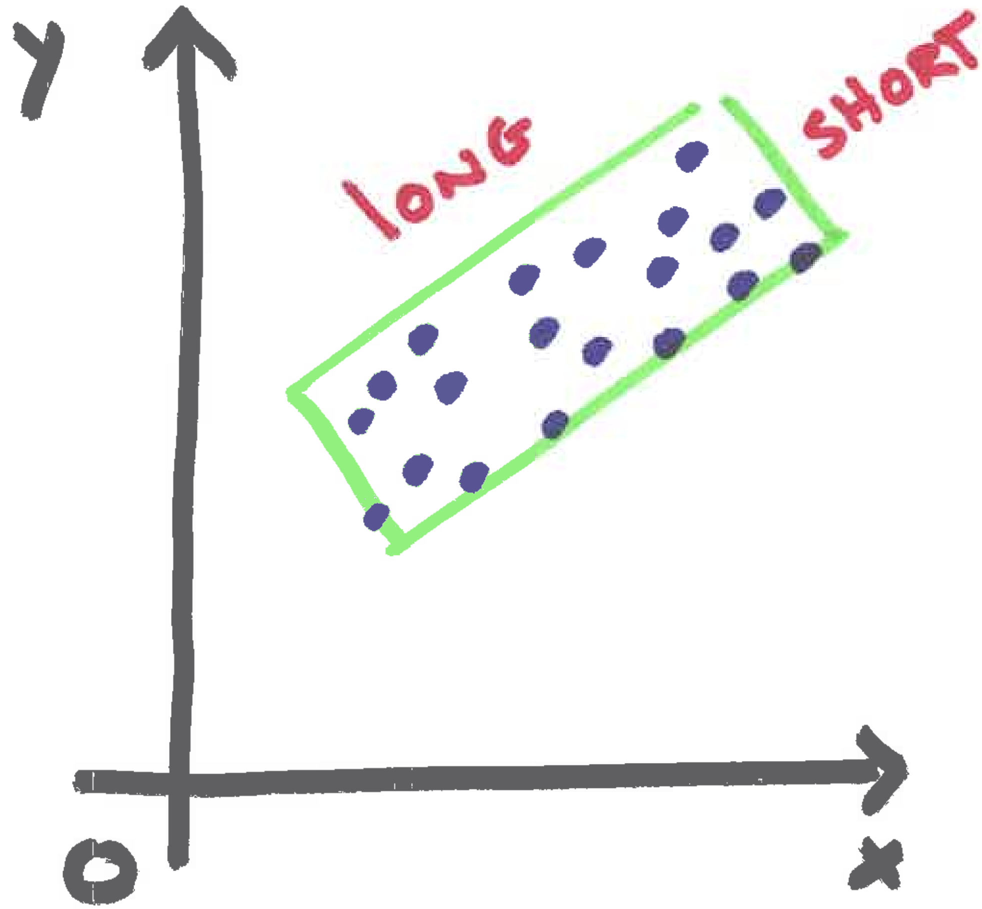
# of scores in each range.



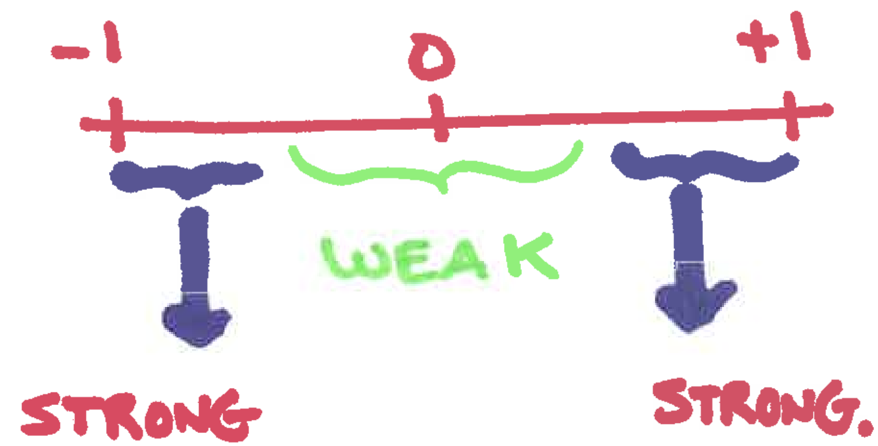
CORRELATION COEFFICIENTS



CORRELATION COEFFICIENTS (CONT.)

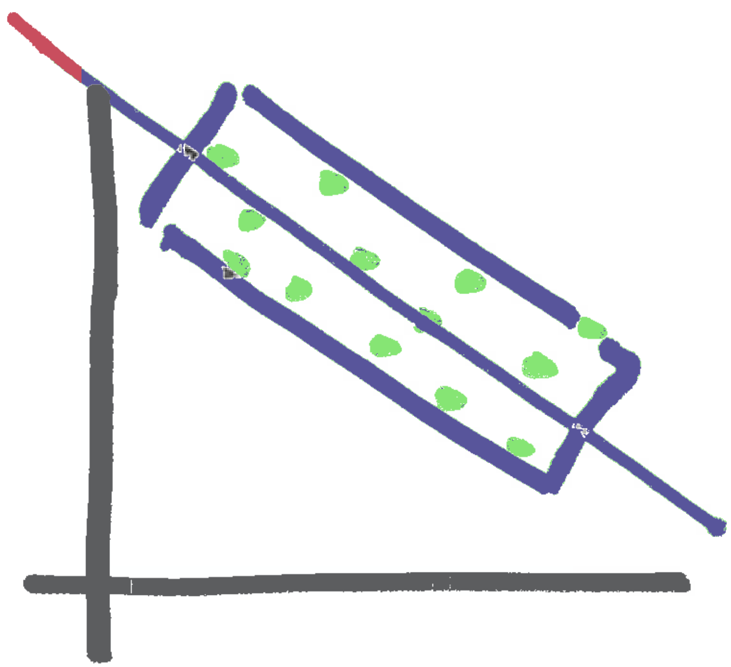


$$r = \pm\left(1 - \frac{\text{SHORT}}{\text{LONG}}\right)$$



REGRESSION LINES

BEST FIT



(1) BUILD THE BOX

(2) DRAW THE BEST FIT LINE

(3) PICK 2 POINTS ON THE LINE.

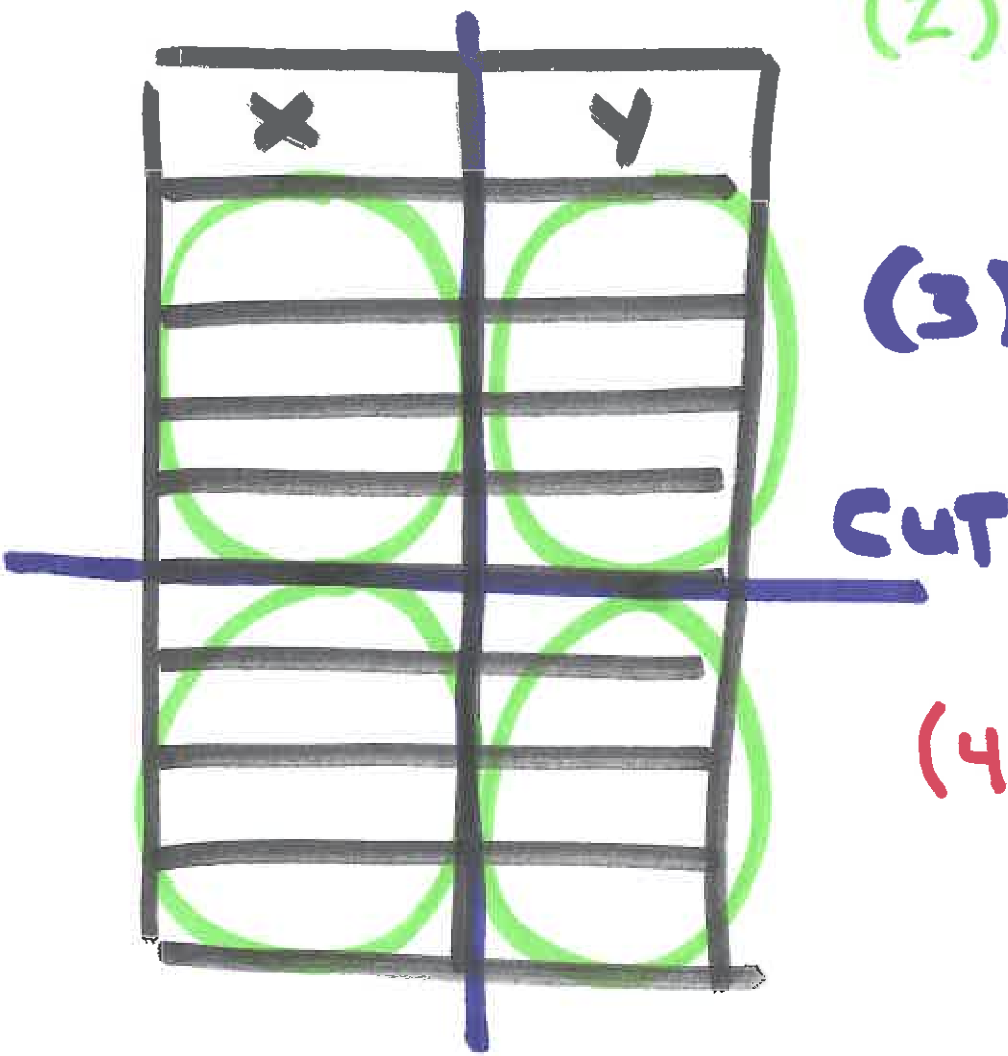
(NOT ACTUAL DATA POINTS)

(4) USE THE POINTS TO BUILD THE RULE.

(5) USE THE RULE TO PREDICT OTHER PTS

REGRESSION LINES

(MEYER)



(1) CUT THE DATA IN HALF

(2) FIND THE AVERAGE VALUES FOR EACH HALF

(3) BUILD A RULE FROM THE AVG. COORDINATES.

(4) USE THE RULE TO PREDICT OTHER POINTS.